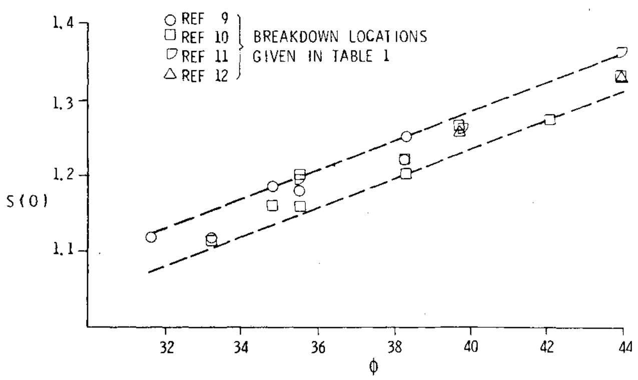

Fig. 1 Correlation for initial value of the swirl parameter for calculating experimentally observed vortex breakdown locations.

Calculation of Breakdown Locations

The experimental data from Refs. 9-12 for vortex breakdown location for flow over thin delta wings in low, subsonic flow is given in Table 1. The spread in the experimental data is usually attributed to slight differences in geometry, particularly at the leading edge (see for example Ref. 13). As indicated in Ref. 1, changes due to Reynolds number variations in these experiments probably do not exceed 10-15%.

It was attempted to match the experimental data using the vortex core model. To provide the edge boundary conditions, the potential flow program was run for the three delta wings at the same angles of attack as listed in Table 1. For all runs, the wing was represented by 25 panels and the free sheet by 40 panels with 5 evenly spaced panels being in the chordwise direction.

As previously indicated, the initial values of the parameters in the axial velocity profile u_0 and u_2 must be specified. For the series of calculations made $u_2(0)=0$ and $u_0(0)$ was varied [$S(0)$ was varied, see Eq. (8)]. It was found that the experimental vortex breakdown locations could be matched by choosing a particular value of $u_0(0)$.

The results are shown in Fig. 1 as a plot of the initial value of the swirl parameter $S(0)$ and the angle ϕ between the wing leading edge and the freestream direction. Each point in the figure represents the initial condition of swirl that results in core breakdown at the same x/l location as an experimental point in Table 1. For the fairly scattered experimental data matched, the points in the figure collapse to a band approximately 0.05 wide in $S(0)$. The correlation of Fig. 1 provides the initial condition required to compute vortex breakdown for delta wings using the quasicylindrical flow model.

References

[1] Hall, M. G., "Vortex Breakdown," *Annual Review of Fluid Mechanics*, Vol. 4, 1972, pp. 195-218.

[2] Kirkpatrick, D.L.I., "Experimental Investigation of the Breakdown of a Vortex in a Tube," RAE Tech. Note No. Aero. 2963, 1965.

[3] Bossel, H. H., "Vortex Equations: Singularities, Numerical Solution, and Axisymmetric Vortex Breakdown," NASA CR-2090, July 1972.

[4] Raat, J., "Vortex Development and Breakdown," AFFDL-TR-75-69, May 1975.

[5] Hall, M. G., "A Numerical Method for Solving the Equations for a Vortex Core," Royal Aircraft Establishment, RAE Tech. Rept. No. 65106, May 1965.

[6] Weber, J. A., Brune, G. W., Johnson, F. T., Lu, P., and Rubbert, P. E., "A Three-Dimensional Solution of Flows over Wings with Leading-Edge Vortex Separation," AIAA Paper 75-866, Hartford, Conn., June 1975.

[7] Johnson, F. T., Lu, P., Brune, G. W., Weber, J. A., and Rubbert, P. E., "An Improved Method for the Prediction of Completely Three-Dimensional Aerodynamic Load Distributions of Configurations with Leading Edge Vortex Separation," AIAA Paper 76-417, San Diego, Calif., July 1976.

[8] Brune, G. W., Weber, J. A., Johnson, F. T., Lu, P., and Rubbert, P. E., "A Three-Dimensional Solution of Flows Over Wings with Leading Edge Vortex Separation," Part I, Engineering Document, NASA CR-132709, Sept. 1975.

[9] Wentz, W. H. and Kohlman, D. L., "Wind Tunnel Investigations of Vortex Breakdown on Slender Sharp-Edged Wings," NASA CR-98737, Nov. 1968.

[10] Earnshaw, P. B. and Lawford, J. A., "Low-Speed Wind-Tunnel Experiments on a Series of Sharp-Edged Delta Wings," Part 1, Royal Aircraft Establishment, RAE Tech. Note No. Aero. 2780, 1961.

[11] Elle, B. J., "An Investigation at Low Speed of the Flow Near the Apex of Thin Delta Wings with Sharp Leading Edges," Aeronautical Research Council, R&M No. 3282, April 1961.

[12] Lawford, J. A. and Beauchamp, A. R., "Low-Speed Wind-Tunnel Measurements on a Thin Sharp-Edged Delta Wing with 70° Leading-Edge Sweep, with Particular Reference to the Position of Leading-Edge-Vortex Breakdown," Aeronautical Research Council, R&M No. 3338, Nov. 1961.

[13] Earnshaw, P. B., "Measurements of Vortex-Breakdown Position at Low Speed on a Series of Sharp-Edged Symmetrical Models," Royal Aircraft Establishment, RAE TR No. 64047, Nov. 1964.

Lightning-Hazard Assessment: A First-Pass Probabalistic Model

Lee McKague*
General Dynamics, Fort Worth, Texas

Introduction

SOME kind of model is required to predict lightning-strike frequency and severity for a new aircraft system. No real model has existed for making such predictions. There also has been no model to help interpret data on lightning strikes to past or existing aircraft systems. Even so, such data has been collected for many years.

Historical evidence, therefore, exists to show the frequency with which aircraft are struck by lightning during flight. This frequency varies widely for various aircraft and service locations. Strike rates have been observed to range from about 2000 hours of service per strike for certain commercial carriers[1] to more than 200,000 hours of service per strike for T-33 jet trainers.[2] Damage records show the fleet distribution of these strikes to the various parts of a given airplane. These records, together with records of ground-based and airborne lightning-strike monitoring stations, provide insight to the severity of strikes.

All of this historical evidence is just that, historical. Predictive application of this evidence to planned aircraft systems relies at present on relatively gross judgments as to whether the new aircraft and mission are like a familiar past or present system.

Therefore, the purpose of this article is to describe a new and fundamental model with which aircraft lightning hazards can be analyzed and predicted. Use of the model for determining lightning-strike frequency of specific aircraft is discussed, and a framework is suggested for extending the use of the model. The model can be used by system planners to study service conditions, strategies, and alternatives that would reduce lightning hazards. It may also be used as a guide to design requirements and as an aid to prediction of service life costs for repair of lightning-strike damage.

Strike Frequency Model

This model considers three basic factors to be involved in calculating the lightning-strike frequency S of an airplane.

Received May 6, 1977.
Index cateories: General Aviation; Military Missions.
*Engineering Specialist, Fort Worth Division.

The first of these is an environmental factor that may be regarded as a lightning-flash density factor F. This flash density factor quantifies the frequency and area density of lightning discharges occurring in the geographic zone(s) of service. The second basic factor is a geometric or design factor termed effective receptor area A, which is a function of aircraft dimensions. This determines the strike rate that will occur when an aircraft of effective area A is placed in a zone of lightning activity having an area density and frequency given by the flash density factor F. The third basic factor is a service-related factor. This is the probability P that service usage conditions will allow the plane to encounter lightning. This factor modifies the strike rate so that the strike frequency S is given by

$$S = F \cdot A \cdot P \tag{1}$$

The first two of these three factors are empirically quantifiable. The probability of encounter P involves judgment to quantify. The accuracy of this quantification can be significantly improved by recognizing that P is a cumulative probability evolved from many factors rather than being a single judgment about service life conditions.

Two fundamental elements of service combine to determine the value of P, and each of these elements can be subdivided into many parts. P_A involves lightning strike occurrences that are not uniformly distributed with respect to altitude. Thusly, this element involves the fraction of lightning discharges occurring in a given altitude band and the fraction of flight time spent within that band. This first element, then, may be regarded as the cumulative altitude probability of encountering lightning based upon an altitude distribution of service time. This element is empirically quantifiable given the altitude distribution of service time. The second element P_M, however, involves more judgment. It is the probability that mission requirements will result in lightning encounter instead of allowing or requiring avoidance.

The value of the net probability of encounter P will result from multiplying the matrices of values for altitude distributions of time and flashes with the matrix of probabilities of encounter determined by the mission(s). Thus, for a multimissioned military fighter, P might be determined by

$$P = [P_A(i,j)]\,[P_M(j,i)] \tag{2}$$

where P_M is the probability of the mission allowing or requiring encounter for mission type j during flight in altitude band i, and P_A is the probability of encounter per altitude band i for mission type j, where P_A is determined from

$$[P_A(i,j)] = \mathrm{diag}[D_1, D_2, \ldots D_i]\,[T(i,j)]$$

$$= \begin{bmatrix} D_1 T_{11} & D_1 T_{12} & \ldots & D_1 T_{1j} \\ D_2 T_{21} & D_{12} T_{22} & \ldots & \\ \vdots & & & \\ D_i T_{i1} & \ldots & & D_i T_{ij} \end{bmatrix} \tag{3}$$

with $\mathrm{diag}[D_1, D_2, \ldots D_i]$ representing the fraction of total lightning flashes occurring within a given altitude band i, and with $[T(i,j)]$ representing the fraction of service time spent in altitude band i for mission type j. For certain commercial carriers there might be but a single mission type, so that $j = 1$.

Quantification

Through satellite observations and other means, it has been observed that approximately 2000 storms are distributed over the world at any given time.[3] The world surface area is approximately 5×10^8 km^2. Therefore, dividing this area into 2000 parts and taking the reciprocal of the square root of area for one part shows that the average linear world storm density s is approximately 2×10^{-3} storm/km. The average distance d across a storm is about 22 km and the average lightning-flash density f within a storm is about 0.36 flashes/km^2·hr.[3] Therefore, the average lightning-flash density factor $\bar{F}$ can be computed to be

$$\bar{F} = s \cdot d \cdot f = 0.016 \text{ flash/km}^2 \cdot \text{hr} \tag{4}$$

However, storms are not uniformly distributed over the earth. A storm concentration factor must be applied to reflect lightning activity in the service zone. This factor results from dividing the typical number of thunderstorm days per year for the service zone by the world average number of thunderstorm days. Using the average distance across a storm d and average world storm density s, the average number of thunderstorm days is calculated to be 16. This duplicates the value determined from world weather records.[4] Hence,

$$F = (n/16)\bar{F} = 0.001\, n \text{ flash/km}^2 \cdot \text{hr} \tag{5}$$

where n represents the number of thunderstorm days per year for the service zone.

Based upon a combination of theory and empiricism, it has been reported that the attractive area A of a ground structure to lightning can be calculated in terms of the structure's height h in meters.[3] For this first-pass model, it has been assumed that the attractive area of a plane in flight would be similar to that of a ground structure when the largest airplane dimension is substituted for structure height. Thus, the attractive area of a plane would be like a saucer with a radius r calculated in terms of the overall fuselage length h according to:

$$r = 0.08\sqrt{h}\,[\exp(-0.02h) - \exp(-0.05h)]$$
$$+ 0.4[1 - \exp(-0.0001h^2)]\,(\text{km}) \tag{6}$$

Consequently, a T-33 jet trainer ($h \approx 12$ m) would have an attractive area A of 0.016 km^2. A moderate-sized commercial carrier such as a Convair 880 or Boeing 727 ($h \approx 40$ m) would have an attractive area of 0.15 km^2.

Quantification of the net probability of encounter P involves a mixture of empiricism and judgment, as discussed in the previous section. It is empirically obvious that storms are not uniformly distributed with respect to altitude. A review of lightning-strike occurrences shows that they also are not uniformly distributed with respect to altitude.[5-7] This review provides the basis for an empirical quantification of the $\mathrm{diag}[D_i]$ matrix, which represents the fraction of total lightning flashes occurring within altitude band i. Values for this matrix are given in Table 1 for 5000 ft (1524 m) altitude bands i. Since these values are a function of the altitude distribution of storms, they are applicable to evaluation of any aircraft system.

The fraction of time spent in each altitude band can be either predicted or determined based upon prior usage. Table 1 also shows the fraction-of-time matrix $[T(i,j)]$ for single-mission ($j = 1$) commercial carriers based upon historical data.[1]

Table 1 Service encounter probability parameters

Altitude[a] band i	Fraction of occurrences D	Flight time fraction T	Mission encounter probability P_M
1	0.20	0.19	0.50
2	0.37	0.24	0.45
3	0.17	0.16	0.40
4	0.09	0.08	0.35
5	0.06	0.07	0.30
6	0.04	0.11	0.25
7	0.03	0.11	0.15
8	0.02	0.03	0.05
9	0.01	0.01	0.01
10	0.01	...	...

[a] Altitude band width = 5000 ft (1524 m).

To illustrate the functioning of the model, a skewed distribution of mission encounter probability $[P_M(i,j)]$ with $j=1$ was chosen based upon judgment. Therefore, it should be noted that P is reasonably insensitive to judgments of P_M. Furthermore, it is likely that errors in judgment will be high for some elements of i and j and lower for others, thereby reducing total error.

The chosen distribution for P_M, shown in Table 1, is inversely related to altitude, implying a reduced ability to avoid storm activity during takeoffs, landings, and holding patterns. Highly turbulent storms would be avoided, of course, but lightning activity is common in rain formations having flight tolerable turbulence. From Table 1 and Eqs. (2) and (3),

$$P=\sum_{i=1}^{n}(D_i)\cdot(T_i)\cdot(P_{Mi})=0.076 \qquad (7)$$

Based upon Eq. (1), the number of service hours per strike S^{-1} for this type of commercial carrier (fuselage length = 40 m, $A=0.15$ km^2) operating in a geographic zone averaging 45 thunderstorm days per year (southeastern USA) would be

$$S^{-1}=(F\cdot A\cdot P)^{-1}=(0.001\times 45\times 0.15\times .076)^{-1}$$
$$=1950 \text{ hr/strike} \qquad (8)$$

This compares quite favorably with observed commercial experience which is commonly accepted to be about 2000 hr/strike.[1]

A similar computation was made for a T-33 jet trainer ($A=0.016$ km^2) based in the Southwest (15 thunderstorm days per year). An altitude distribution of flight time for a fighter was used, and a judgment was made regarding mission probability of encounter. A skewed encounter distribution was used here also, but the maximum P_M, for altitudes to 5000 ft ($i=1$), was only 0.20 instead of the 0.50 value used for commercial carriers. This reflects the flexibility of flight schedules and paths available for training that are denied to scheduled airlines. The net probability of encounter P was calculated to be 0.017. Thus, for the assumed trainer situation, the predicted service rate is

$$S^{-1}=(F\cdot A\cdot P)^{-1}=(0.001\times 15\times 0.016\times 0.017)^{-1}$$
$$=245{,}000 \text{ hr/strike} \qquad (9)$$

which compares favorably with documented T-33 trainer experience which is 222,000 hours of service per strike.[2] This result, together with the result for commercial carriers, shows that the model can describe the range of observed experience. Therefore, the model can be used to analyze existing data. It can also be applied to the design and service planning of new systems.

Applications

Planners who must develop service usage strategies can benefit from studies to quantify the net probability of encounter P. The model can also benefit those seeking to predict life-cycle repair costs. These costs will be a function of the number of strikes per fleet per life, the distribution of these strikes to the various aircraft parts, and the severity of the strikes.

The number of strikes per fleet per life will be the product of the strike rate S, the service life in hours, and the number of planes in the fleet. The distribution of the fleet total of strikes to the various aircraft parts will probably vary with aircraft size and speed. Distributions can be estimated based upon previous aircraft experience.[2,7-9] As an example, Table 2 shows the empirical probability of strike to a particular part of a fighter such as an F-4. Using such a distribution, the severity of strikes to a particular component can then be estimated on a probabilistic basis by obtaining data on the occurrence rate of lightning having certain levels of current. Such data has been collected for ground-based recording stations[3] and includes distributions of duration and time between re-strikes. Time-related data and flight speed can be used to predict swept stroke behavior.

Table 2 Probability of strike to a particular part of a fighter

Component	Probability
Pitot/radome	0.39
Wing/wingtip	0.25
Empennage	
Vertical fin	0.12
Horizontal fin	0.01
Horizontal stabilizer	0.05
Rudder	0.04
Other	0.01
Fuselage	0.05
Canopy	0.05
Miscellaneous	
Gear doors	0.01
External stores	0.01
Antenna	0.01

Such analyses do not preclude the possibility of strikes having greater severity than that indicated by the probability analysis. However, the analyses can provide a reasonable basis for estimating service strike occurrences and their likely distribution and severity. This, in turn, may provide the basis for making protection/no-protection decisions for certain components during design of new systems.

References

[1] Fitzgerald, D. R., "USAF Flight Lightning Research," *Lightning and Static Electricity Conference*, Air Force Aerodynamics Lab., AFAL-TR-68-290 Part II, 1968, pp. 123-134.

[2] Cummings, L. E., "A Review and Analysis of USAF Operational Lightning Problems," *Lightning and Static Electricity Conference*, AFAL-TR-71-89, 1970, pp. 1-14.

[3] Cianos, N. and Pierce, E. T., "A Ground-Lightning Environment for Engineering Usage," Stanford Research Institute, Menlo Park, Calif., SRI Project 1834, Aug. 1972.

[4] Brooks, C.E.P., "The Distribution of Thunderstorms Over the Globe," Air Ministry Meterological Memoirs, No. 24, H. M. Stationery Office, London, 1925.

[5] Perry, B. L., "British Researches and Protective Recommendations of the British Air Registration Board," *Lightning and Static Electricity Conference*, AFAL-TR-68-290 Part II, 1968, pp. 81-103.

[6] Appleman, H. S., "Lightning Hazard to Aircraft," HQ Air Weather Service (MAC), USAF, Tech. Rept. 179, April 1971.

[7] "Analysis of USAF Jet Aircraft Experiencing Lightning Strikes," Los Angeles Division of North American Rockwell, NA-71-636, 1971.

[8] Schwartz, H. S. and Austin, C. R., "Effects of Lightning Strikes on Fiber-Reinforced Structural Plastic Composites," Air Force Materials Lab., AFML-TR-68-196, 1968.

[9] Shaeffer, J. F. and Weinstock, G. L., "Aircraft Related Lightning Mechanisms," AFAL-TR-72-386, 1972.